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II. LOGICAL DEDUCTIONS FROM THE HYPOTHESIS THAT THE ANGLE SUM IS LESS THAN TWO RIGHT ANGLES.

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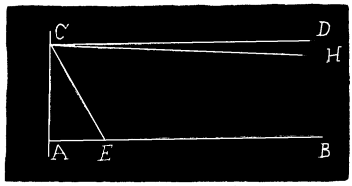
Erect the perpendiculars AB and CD to the straight line AC at the points A and C, On AB lay off $AE=AC$ and draw a straight line from C to E.

By construction the triangle ACE is isosceles. Hence, the angle $AEC=ACE$.



By hypothesis the angle sum of every triangle and, hence, of ACE is supposed to be less than two right angles. In accordance with this assumption let the angle sum of the triangle ACE be equal to two right angles$-a$. Construct $DCH=a$. Then $CAE+ACE+AEC=CAE+ACH$.

Subtract $CAE+ACE$ from both members. Then $AEC=ECH$.

But AEC and ECH are alternate angles. Hence, CH is parallel to AB in the Euclidian sense, that is, it will not meet AB however far both lines may be produced. Euclid. Book I. Proposition XXVII.

Therefore, when the angle sum is assumed in any instance to be equal to $CAE+ACH$, the line CH can not consistently with that hypothesis meet AB.

If, however, the angle sum is assumed to be greater than $CAE+ACH$, it is consistent with this hypothesis to suppose that the line CH may meet AB. For if we make this supposition a triangle will be formed whose angle sum is greater than $CAE+ACH$.

Further, it is inconsistant with the hypothesis to suppose that the line CH can not meet AB. For to deny that CH can meet AB is to deny that a triangle whose angle sum is greater than $CAE+ACH$ can be formed, which is to deny the hypothesis.

But the deduction that CH may meet AB contradicts the conclusion that CH can not meet AB. Therefore, the hypothesis that the angle sum may be greater than $CAE+ACH$ is inconsistant with the hypothesis that the angle sum in any instance is equal to $CAE+ACH$.

That is, the hypothesis that the angle sum is a variable less than two right angles approaching two right angles as a limit is contradictory and hence absurd.

If the hypothesis that the angle sum is less than two right angles is false, sound science requires that the logical deductions from the hypothesis should likewise be false.

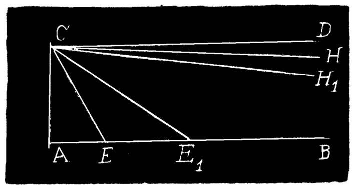
One deduction is that the lines AB and CH making angles with AC whose sum is less than two right angles do not meet. This contradicts Euclid's axiom 12.

Another deduction is that the alternate angles AEC and ECD are not equal although the perpendiculars AB and CD to AC are parallel to each other and can not meet.

Another deduction is that through the same point two straight lines may be drawn parallel to the same straight line. This contradicts the statement known as Playfair's axiom.

Still another of these deductions is that if one side of a triangle be produced the exterior angle is greater than the sum of the two interior and opposite angles.

Lay off $EE_1 = CE$ and draw a straight line from C to E_1.



By hypothesis $ECE_1 + EE_1C + CEE_1 < 2$ right angles. But $AEC + CEE_1 = 2$ right angles. Hence $ECE_1 + EE_1C + CEE_1 < AEC + CEE_1$, and $ECE_1 + EE_1C < AEC$.

Add $EAC + ACE$ to both members of this inequality. Then $E_1AC + ACE_1 + AE_1C < EAC + ACE + AEC$. That is, the angle sum of ACE_1 is less than that of ACE.

Let the angle sum of $ACE_1 = 2$ right angles$-b$. But the angle sum of $ACE = 2$ right angles$-a$.

Hence, $b > a$.

Construct $DCH_1 = b$.

Then $CAE_1 + ACE_1 + AE_1C = CAE_1 + ACH_1$, in which $ACH_1 < ACH$.

Subtract $CAE_1 + ACE_1$ from both members. Then $AE_1C = E_1CH_1$.

But AE_1C and E_1CH_1 are alternate angles. Hence, CH_1 is parallel to AB in the Euclidian sense, that is, it will not meet AB however far both lines may be produced. Euclid. Book I. Proposition XXVII.

Therefore, when the angle sum is assumed in any instance to be equal to $CAE_1 + ACH_1$, the line CH_1 can not consistently with that hypothesis meet AB.

The angle sum of ACE is assumed to be $CAE + ACH$, that is, greater than $CAE_1 + ACH_1$.

If greater, the line CH_1 may consistently with the hypothesis meet AB.

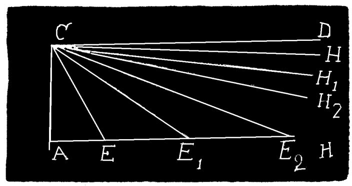
But the deduction that CH_1 may meet AB contradicts the conclusion already reached that CH_1 can not meet AB.

Therefore, the hypothesis that the angle sum may be greater than $CAE_1 + ACH_1$ is inconsistent with the hypothesis that the angle sum in any instance is equal to $CAE_1 + ACH_1$. That is, the hypothesis that the angle sum is a variable less than two right angles approaching two right angles as a limit in value is a contradiction and is therefore false.

Let us proceed with our investigation. Construct the successive isosceles triangles CE_1E_2, CE_2E_3,&c.

Let us consider the series of triangles AE_2C, AE_2C,&c. From the hypothesis that the angle sum is less than two right angles, the conclusion is reached by a process used above in this article that the angle sum of each triangle is less than that of the preceeding triangles in the series.

Draw the lines CH_2, CH_3,&c., making $CAB+ACH_2$ equal to the angle sum of the triangle AE_2C, and $CAB+ACH_3$ equal to the angle sum of the triangle AE_3C.&c. It then follows that $DCH_1<DCH_2$ and $DCH_2<DCH_3$,&c.



These results contradict Euclid. They seem also to be inconsistent with each other, for they apparently teach that the lines CH, CH_1, CH_2, CH_3,&c., both do and do not meet AB. Furthermore the inconsistency is interminable inasmuch as the series DCH, DCH_1, DCH_2, DCH_3,&c., is non-terminating.

Lobatschewsky in enunciating his doctrine of "Imaginary Geomotry" expressly calls his triangle "rectilineal." The logical, geometrical and metaphysical difficulties that follow the denial of the Euclidian axiom 12 and the Euclidian angle sum are so great, however, that non Euclidian writers are now maintaining that Lobatschewsky's triangle can not be drawn in a Euclidian plane and that it is not in fact rectilineal. Since this homeless, outcast triangle is unable to find a "local habitation" in the space of the Alexandrian geometer, the non-Euclidians have excogitated a space especially to contain it called by them "pseudo spherical." Helmholtz in his Lecture "On the origin and significance of geometrical axioms" refers to a "pseudo spherical surface" as "saddle-shaped." He says that the Italian Mathematician E. Beltrami investigated its properties and gave it the name pseudo spherical. Later on in his Lecture he dexterously passes from the phrase—"pseudo spherical surface" to pseudo spherical space." This performance is plainly psendological. Surface manifestly is not identical with space. Surfaces may be located in space but should not be confounded with space. Beltrami contributes to modern geometrical literature the expression "pseudospherical surface." Helmholtz treats it as identical with "pseudo spherical space" by pseudo logical reasoning and pseudo philosophical speculation.